THE METHOD OF SIMULATED MAXIMUM LIKELIHOOD FOR THE ESTIMATION OF DYNAMIC ORDERED PROBIT: AN APPLICATION TO COUNTRY-RISK FOR NON-DEVELOPED COUNTRIES

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Abstract
This paper aims to give a detailed explanation of the econometric methodology necessary to estimate dynamic probit models with ordinal dependent variables. A typology of cases are established which appear when considering different choices of individual heterogeneity along with time correlation. To be able to estimate by maximum likelihood the models which come out of the different alternatives proposed, simulation techniques are used and put into practice by the GHK simulator and, in this way, estimators by simulated maximum likelihood are obtained. Finally, all the models described are used to measure and determine the macroeconomic factors which explain the ratings of country-risk in non-developed countries.

Keywords: Country risk, panel data, external debt, dynamic ordered probit.
JEL: C33, C35, F34, H63, O16

1. Introduction

Over the last few years we have witnessed an explosion in the application of panel data applied to dynamic models with qualitative dependent variables. In this area the work of Börsch-Supan, Hajivassiliou, Kotlikoff and Morris (1990); Arellano and Bover (1997), Inkmann (1999), Train (2003) and Waelbroeck (2003) stand out. However, the majority of these studies deal with qualitative dependent variables and with binomial or multinomial distribution. In this paper the relevant econometric methodology is developed to estimate dynamic models with an ordinal dependent variable; these models come from the panel data probit models allowing for the

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existence as much of individual heterogeneity as of the temporal correlation within each individual. The estimation technique outlined, which is suitable for the complete typology of the models under consideration, is the simulated maximum likelihood employing the habitual GHK simulator as that allows for the specification of different alternatives, as much of individual effects as of dynamic correlations in the variance-covariance matrices of disturbances.

However, there are other estimation alternatives, such as the simulated moment method (Inkmann, 1999), or the bayesian methods Waelbroeck (2003), the implementation of which, in the cases considered, is less direct. The development of the econometric formalisation of the dynamic ordered probit is applied to a case study which refers to the establishment of the main explanatory macroeconomic factors of the country-risk ratings for non-developed countries. In these countries there are no markets with sufficient liquidity to negotiate their debt, which is why it could be interesting to put forward a model which allow for the calculation of risk and, consequently, its evolution over the last few years. This model would also be able to calculate the sensitivities of the probabilities associated with the different qualifications with regard to the main macroeconomic factors, thus enabling the quantifying of the influence of each one.

Finally, comparisons between the statistical or unconditional probabilities versus the conditional probabilities, obtained if the existing inertia of the evolution of each country is borne in mind, could be established. This work has been organised in the following manner: section 2 develops the model and also the econometric methodology necessary for its estimation. In section 3 an empirical application is carried out, which proposes both a measurement and an explanation of the dynamic evolution of the country risk in non-developed countries from a sample obtained from the database of the World Bank. Finally, section 4 summarises the main conclusions of the work carried out.

2. The method SML for the estimation a Dynamic Ordered Probit

To suitably specify the rating model, we assume that each rating decision for each country at each point in time depends increasingly on a random index function $V^*_{i,t}$ which related a continuous measurement of underlying utility, as a dependent variable, to the value of the macroeconomic variables as regressors. If we assume a linear specification, the random index equation will be given by the usual regression model:

$$\begin{aligned} &V^*_{i,t} = \beta^*_0 + \beta^*_1 x_{1,i,t} + \ldots + \beta^*_k x_{k,i,t} + u^*_{i,t} = X^T_{i,t} \cdot \beta + u^*_{i,t} \\ &u^*_{i,t} \text{ i.i.d.} \left(0, \sigma^2_{u^*}\right) \\ &i = 1, \ldots, N \\ &t = 1, \ldots, T \end{aligned} \tag{2.1}$$

The only difference from the regression model is that the dependent variable is not observable. The rating decision observed will depend on whether the non-observable value exceeds certain fixed thresholds, thus:

$$Y_{i,t} = \begin{cases} 0 & \text{if} \quad V^*_{i,t} < \gamma^*_0 \\ 1 & \text{if} \quad \gamma^*_0 \le V^*_{i,t} < \gamma^*_1 \\ 2 & \text{if} \quad \gamma^*_1 \le V^*_{i,t} < \gamma^*_2 \\ \vdots & \qquad \vdots \\ J & \text{if} \quad V^*_{i,t} \ge \gamma^*_{J-1} \end{cases} \tag{2.2}$$

with $\gamma^*_0 \le \gamma^*_1 \le \gamma^*_2 \le \cdots \le \gamma^*_{J-1}$. Therefore, the rating, observed as an ordinal variable, depends on the position of the random utility. We can then map a quantitative, but non-observable, variable on an observed ordinal variable. The value of the random index has a systematic component depending on $x_{i,t}$ variables and a random component included in term of disturbance $u^*_{i,t}$. When specifying a random distribution for $u^*_{i,t}$ we obtain the models usually found in the literature [see Arellano and Bover, 1997]. Logistic distribution and normal distribution are usually employed. To identify the model we have to perform normalisation of origin subtracting γ^*_0 from both sides of equation (2.1), thus establishing the new minimum value of the threshold as 0. We also have to normalise the scale dividing the entire equation by σ_{u^*}, so that the noise of the normalised equation has unitary variance. The relation between the parameters and variables of the initial and the normalised models is given by Train(2003):

$$
\begin{aligned}
&\forall j = 1,\cdots,J-1 \quad \gamma_j = \frac{\gamma_j^* - \gamma_0^*}{\sigma_{u^*}} \to \gamma_0 = 0 \\
&\forall k = 1,\cdots,K \quad \beta_0 = \frac{\beta_0^* - \gamma_0^*}{\sigma_{u^*}} \quad \beta_k = \frac{\beta_k^*}{\sigma_{u^*}} \\
&V_{i,t} = \frac{V_{i,t}^* - \gamma_0^*}{\sigma_{u^*}} \\
&u_{i,t} = \frac{u_{i,t}^*}{\sigma_{u^*}}
\end{aligned}
\tag{2.3}
$$

With the previous operations, the model is described by the following equations:

$$
\begin{aligned}
&V_{i,t} = \beta_0 + \beta_1 x_{1,i,t} + \ldots + \beta_k x_{k,i,t} + u_{i,t} = X_{i,t}^T \cdot \beta + u_{i,t} \\
&u_{i,t} \text{ i.i.d.} \left(0,1\right) \\
&i = 1,\ldots,N \quad t = 1,\ldots,T \quad k = 1,\ldots,K
\end{aligned}
\tag{2.4}
$$

And

$$
Y_{i,t} = \begin{cases} 0 & if \quad V_{i,t} < 0 \\ 1 & if \quad 0 \le V_{i,t} < \gamma_1 \\ 2 & if \quad \gamma_1 \le V_{i,t} < \gamma_2 \\ \vdots & \qquad \vdots \\ J & if \quad V_{i,t} \ge \gamma_{J-1} \end{cases}
\tag{2.5}
$$

$$
0 = \gamma_0 \le \gamma_1 \le \cdots \le \gamma_{J-1}
$$

This normalisation does not affect the observed rating, since the random utility values correspond to the same section as before, whereas the other parameters are obtained.

From equations (2.4) and (2.5) above, the probability of each rating observed, for each individual I and moment in time t can be expressed as:

$$
\begin{aligned}
&\Pr\left(Y_{i,t} = 0\right) = \Pr\left(V_{i,t} < 0\right) = \Pr\left(u_{i,t} < -X_{i,t}{'}\beta\right) \\
&\vdots \\
&\Pr\left(Y_{i,t} = j\right) = \Pr\left(\gamma_{j-1} \le V_{i,t} < \gamma_j\right) = \Pr\left(\gamma_{j-1} - X_{i,t}{'}\beta \le u_{i,t} < \gamma_j - X_{i,t}{'}\beta\right) \\
&\vdots \\
&\Pr\left(Y_{i,t} = J\right) = \Pr\left(\gamma_{J-1} \le V_{i,t}\right) = \Pr\left(\gamma_{J-1} - X_{i,t}{'}\beta \le u_{i,t}\right)
\end{aligned}
\tag{2.6}
$$

Equation (2.6) shows that, given the observed values of the $\boldsymbol{X_{i,t}}$ variables, the probability of each choice will depend on the value of parameters β_k (k = 1, …, K) accompanying the regressors, on thresholds γ_j (j = 1, …, J-1) and the distribution function assumed for noise $u_{i,t}$. With regards to t he latter, we have to remember that, since the model includes panel data (the sample corresponds to a set of countries observed for a certain period of time), although the noise has been normalised, the structure of the variance-covariance matrix of the disturbances may be relatively complex if random heterogeneity effects and time dependences by individual are permitted. If, in this case, the disturbances of the random utility equation $\varepsilon \sim N(0,\Omega)$ are expressed as (2.4), we obtain the Ordered Probit model for panel data [Cheung, 1996; Hausman et al, 1991]. The final formulation of this model will depend on the hypotheses considered concerning matrix Ω. The different types considered in this study are similar to those used in [Ackerber, 1999; Berg and Coke, 2004; Hajivassiliou and McFadden, 1990]:

$$1.\text{-} \begin{cases} \varepsilon_{i,t} = u_{i,t} \\ u_{i,t} \text{ i.i.d. } N(0,1) \end{cases}$$

In this case, we assume that there are no individual random heterogeneity effects or time correlations. Therefore, given that $var(u_{i,t})$=1, matrix Ω will be the identity matrix and the resulting model will be equivalent to an Ordered Probit with cross section data.

$$2.\text{-} \begin{cases} \varepsilon_{i,t} = \alpha + u_{i,t} \\ \alpha \overset{d}{=} N(0, \sigma_\alpha^2) \\ u_{i,t} \; i.i.d. \; N(0,1) \end{cases}$$

There is now an individual random effect α for each country, although the variance of the random effect is always the same. Therefore, the variance of the disturbances will remain constant and the covariances between the disturbances of the same individual will not be zero; in other words,

$$\forall i,t \quad \text{var}(\varepsilon_{i,t}) = \sigma_\alpha^2 + 1$$

$$\text{cov}(\varepsilon_{i,t}, \varepsilon_{j,s}) = \begin{cases} \sigma_\alpha^2 & i = j \quad t \neq s \\ 0 & i \neq j \quad \forall t,s \end{cases}$$

With this specification, matrix $\Omega_{(TN)x(TN)}$ will be with diagonal blocks[1], thus:

$$\Omega = \begin{pmatrix} \sum_1 & 0 & \cdots & 0 \\ 0 & \sum_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & \sum_N \end{pmatrix}$$

where each sub-matrix of dimension TxT on the principal diagonal has the following structure:

$$\sum_i = \begin{pmatrix} \sigma_\alpha^2 + 1 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\alpha^2 + 1 & \cdots & \sigma_\alpha^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 + 1 \end{pmatrix}$$

3.- $\begin{cases} \varepsilon_{i,t} = \rho \varepsilon_{i,t-1} + u_{i,t} \\ \rho \in (-1,1) \\ u_{i,t} \text{ i.i.d. } N(0,1) \end{cases}$

In this case, there is time correlation generated by an AR(1) with the same parameter ρ in all individuals. Matrix Ω is with diagonal blocks again and the sub-matrices on the principal diagonal have the usual structure of disturbances in first order regressive models:

$$\sum_i = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & \rho & \cdots & \rho^{T-1} \\ \rho & 1 & \cdots & \rho^{T-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \cdots & 1 \end{pmatrix}$$

4.- $\begin{cases} \varepsilon_{i,t} = \alpha_i + u_{i,t} \\ \alpha_i \overset{d}{=} N(0, \sigma_{\alpha_i}^2) \\ u_{i,t} \; i.i.d. \, N(0,1) \end{cases}$

This is similar to case 2, but diversity between the random effects of the different individuals is now permitted, so the structure of matrix Ω is identical to type 2 but with a different parameter $\sigma_{\alpha_i}^2$ in each sub-matrix $\sum_i$.

[1] For all the cases considered, matrix Ω will always have a diagonal blocks structure. This is particularly important to provide the maximum likely estimation of the model, since, as we will see later, it enables us to additively separate the logarithm from individual likelihoods.

$$5.-\begin{cases}\varepsilon_{i,t}=\rho_i\varepsilon_{i,t-1}+u_{i,t}\\ \rho_i\in(-1,1)\\ u_{i,t}\ \text{i.i.d. } N(0,1)\end{cases}$$

In this option, the dynamics may be different for each individual and the structure of matrix Ω is similar to type 3, but with a different parameter ρ_i in each sub-matrix $\sum_i$.

$$6.-\begin{cases}\varepsilon_{i,t}=\alpha+\xi_{i,t}\\ \alpha\overset{d}{=}N(0,\sigma_\alpha^2)\\ \xi_{i,t}=\rho\xi_{i,t-1}+u_{i,t}\\ \rho\in(-1,1)\\ u_{i,t}\ i.i.d.\ N(0,1)\end{cases}$$

This case permits the existence of both a random effect and a time correlation for each individual, although parameters σ^2_α and ρ must be the same for all the individuals. Matrix Ω is in diagonal blocks again, and each element of the principal diagonal is given by:

$$\sum_i=\begin{pmatrix}\sigma_\alpha^2+\frac{1}{1-\rho^2} & \sigma_\alpha^2+\frac{\rho}{1-\rho^2} & \cdots & \sigma_\alpha^2+\frac{\rho^{T-1}}{1-\rho^2}\\ \sigma_\alpha^2+\frac{\rho}{1-\rho^2} & \sigma_\alpha^2+\frac{1}{1-\rho^2} & \cdots & \sigma_\alpha^2+\frac{\rho^{T-2}}{1-\rho^2}\\ \vdots & \vdots & \ddots & \vdots\\ \sigma_\alpha^2+\frac{\rho^{T-1}}{1-\rho^2} & \sigma_\alpha^2+\frac{\rho^{T-2}}{1-\rho^2} & \cdots & \sigma_\alpha^2+\frac{1}{1-\rho^2}\end{pmatrix}$$

$$7.-\begin{cases}\varepsilon_{i,t}=\alpha_i+\xi_{i,t}\\ \alpha_i\overset{d}{=}N(0,\sigma_{\alpha_i}^2)\\ \xi_{i,t}=\rho\xi_{i,t-1}+u_{i,t}\\ \rho\in(-1,1)\\ u_{i,t}\ i.i.d.\ N(0,1)\end{cases}$$

This case generalises type 6, permitting heterogeneity between both the random effects and the time correlations of each individual. Matrices $\sum_i$ of the principal diagonal will be similar to the above, but with different parameters $\sigma^2_{\alpha_i}$ and ρ_i for each individual.

The set of alternatives considered above is not exhaustive and different variance-covariance schemes can be implemented, giving rise to different matrix Ω structures. However, we believe that the alternatives considered are broad enough to contemplate the individual and time dependencies observed in actual cases. One interesting generalisation would be to permit the existence of nonzero correlation between the random effects of different individuals. However, in this case the diagonal block structure of matrix Ω would be broken and the computational estimation problems increase enormously. To estimate parameters β and γ, and those included in the covariance matrix Ω, we have to maximise the likelihood function logarithm expressed, under the hypothesis of normality, as[2]:

$$\ln L(\beta,\gamma,\Omega|Y,X) = \sum_{i=1}^{N} \ln \Pr\left(Y_{i,1} = j_{i,1}, \cdots ; Y_{i,T} = j_{i,T}\right) =$$
$$\sum_{i=1}^{N} \ln \Phi\left(\gamma_{j_{i,1}-1} - X'_{i,1}\beta \leq \varepsilon_{i,1} \leq \gamma_{j_{i,1}} - X'_{i,1}\beta, \cdots ; \gamma_{j_{i,T}-1} - X'_{i,T}\beta \leq \varepsilon_{i,T} \leq \gamma_{j_{i,T}} - X'_{i,T}\beta\right) \tag{2.6}$$

Where the choices $j_{i,t}$ of the i-th individual belong to the set of alternatives (j=0, …, J) and parameters $\gamma_{i,t}$ are included in the threshold vector (0, $\gamma_1, \ldots, \gamma_{J-1}$). Function Φ(·) is the normal multivariate distribution, so we have to consider that, if the covariance matrix is between types 2 to 7, the likelihood function requires calculating normal multidimensional distribution integrals the dimension of which grows[3] with T. To evaluate the multidimensional integrals of the likelihood function, we need to use simulation methods, of which the most commonly used in this context is the GHK simulator[4] [Börsch-Supan and Hajivassiliou, 1993; Hajivassiliou and McFadden, 1990; Hajivassiliou et al, 1996; Inkmann, 1999; Train, 2003; Börsch-Supan Waelbroeck, 2003].

The simulator starts with the Cholesky decomposition of the positive definite matrix $\Omega = C \cdot C'$ where C is an inferior triangular matrix. Due to the diagonal block structure of matrix Ω, matrix C can

[2] The log-likelihood can be considered by individual, since there are no correlations between them. However it cannot be broken down over time since there may be time correlations within each individual. This form of likelihood is a direct consequence of the diagonal block structure of matrix Ω, with non-diagonal $\sum_i$ sub-matrices.

[3] If matrix Ω was not in diagonal blocks due to the existence of correlations between individuals, the normal multidimensional integral would be in the order of NxT which, on this level, is of an impossible to solve computational complexity.

[4] There are other alternative simulation methods such as those described in, among others [Börsch-Supan et al, 1990; Breiung and Lechner, 1998; Chib and Greenber, 1996; Fleming and Mae, 2002; Geweke et al, 1994; Green, 2002; Honoré, 2002].

be constructed from the Cholesky decomposition of each principal sub-matrix $\sum_i = C \cdot C'$. In this case, C is a diagonal block matrix with C_i sub-matrices on the principal diagonal. The disturbance vector ε_i for each individual can then be expressed as $\varepsilon_i = C_i \cdot \eta_\iota$ with $\eta_\iota \sim N(0,I)$. Since C_i is inferior triangular, the elements of the disturbance vector are obtained recursively for all moments in time $t = 1,\ldots,T$ from the standardised random variables:

$$
\begin{aligned}
\varepsilon_{i,1} &= c^i_{1,1}\eta_{i,1} \\
\varepsilon_{i,2} &= c^i_{2,1}\eta_{i,1} + c^i_{2,2}\eta_{i,2} \\
&\vdots \\
\varepsilon_{i,T} &= c^i_{T,1}\eta_{i,1} + c^i_{T,2}\eta_{i,2} + \cdots + c^i_{T,T}\eta_{i,T}
\end{aligned}
$$

The decomposition in question would have the following form (matrix C_i is inferior triangular):

$$
\begin{pmatrix}
c^i_{1,1} & 0 & \cdots & 0 \\
c^i_{2,1} & c^i_{2,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
c^i_{T,1} & c^i_{T,2} & \cdots & c^i_{T,T}
\end{pmatrix}
\bullet
\begin{pmatrix}
\eta_{i,1} \\ \eta_{i,2} \\ \vdots \\ \eta_{i,T}
\end{pmatrix}
=
\begin{pmatrix}
\varepsilon_{i,1} \\ \varepsilon_{i,2} \\ \vdots \\ \varepsilon_{i,T}
\end{pmatrix}
$$

The log-likelihood function of equation (2.7) can be decomposed as:

$$
\begin{aligned}
L(\beta,\gamma,\Omega|Y) = \sum_{i=1}^{N} \ln \Phi &\left[\left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^i_{1,1}}\right) \le \eta_1 < \left(\frac{\gamma_{j_{i,1}} - X'_{i,1}\beta}{c^i_{1,1}}\right)\right] \cdot \\
\Phi &\left[\left(\frac{\gamma_{j_{i,2}-1} - X'_{i,2}\beta}{c^i_{2,2}}\right) \le \eta_2 < \left(\frac{\gamma_{j_{i,2}} - X'_{i,2}\beta}{c^i_{2,2}}\right) \middle| \left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^i_{1,1}}\right) \le \eta_1 < \left(\frac{\gamma_{j_{i,1}} - X'_{i,1}\beta}{c^i_{1,1}}\right)\right] \cdot \\
\Phi &\left[\left(\frac{\gamma_{j_{i,3}-1} - X'_{i,3}\beta}{c^i_{3,3}}\right) \le \eta_3 < \left(\frac{\gamma_{j_{i,3}} - X'_{i,3}\beta}{c^i_{3,3}}\right) \middle| \left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^i_{1,1}}\right) \le \eta_1 < \left(\frac{\gamma_{j_{i,1}} - X'_{i,1}\beta}{c^i_{1,1}}\right) \wedge \left(\frac{\gamma_{j_{i,2}-1} - X'_{i,2}\beta}{c^i_{2,2}}\right) \le \eta_2 < \left(\frac{\gamma_{j_{i,2}} - X'_{i,2}\beta}{c^i_{2,2}}\right)\right] \cdots
\end{aligned}
\tag{2.7}
$$

We can now evaluate likelihood per individual, for any value of β, γ and C, simulating the truncated normal distributions and evaluating the product of one-dimensional integrals of the normal distribution. This can be done in the following stages for each individual:

1.- For each individual, calculate $\left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^i_{1,1}}\right) \le \eta_{i,1} < \left(\frac{\gamma_{j_{i,1}} - X'_{i,1}\beta}{c^i_{1,1}}\right)$.

2.- Simulate a value $\eta^{r}_{i,1}$ of the truncated univariate normal distribution of $\eta_{i,1}$. To do this we can make:

- Simulate a value $\mu_{i,1}$ of a uniform distribution in [0,1].
- Calculate

$$\eta^{r}_{i,1} = \Phi^{-1}\left\{\mu^{r}_{i,1}\left[\Phi\left(\frac{\gamma_{j_{i,1}} - X'_{i,1}\beta}{c^{i}_{1,1}}\right) - \Phi\left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^{i}_{1,1}}\right)\right] + \Phi\left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^{i}_{1,1}}\right)\right\}$$

3.- Calculate $\Phi\left[\frac{\gamma_{j_{i,2}-1} - X'_{i,2}\beta - c^{i}_{2,1}\eta_{i,1}}{c^{i}_{2,2}} \leq \eta_{i,2} < \left(\frac{\gamma_{j_{i,2}} - X'_{i,2}\beta - c^{i}_{2,1}\eta_{i,1}}{c^{i}_{2,2}}\right)\middle| \eta_{i,1} = \eta^{r}_{i,1}\right]$.

4.- Simulate a value $\eta_{i,2}$ of the corresponding truncated univariate normal distribution using the same processes as before:

- Simulate a value $\mu_{i,2}$ of a uniform distribution [0,1].
- Calculate

$$\eta^{r}_{i,2} = \Phi^{-1}\left\{\mu^{r}_{i,2}\left[\Phi\left(\frac{\gamma_{j_{i,2}} - X'_{i,2}\beta - c^{i}_{2,1}\eta^{r}_{i,1}}{c^{i}_{2,2}}\right) - \Phi\left(\frac{\gamma_{j_{i,2}-1} - X'_{i,2}\beta - c^{i}_{2,1}\eta^{r}_{i,1}}{c^{i}_{2,2}}\right)\right] + \Phi\left(\frac{\gamma_{j_{i,2}-1} - X'_{i,2}\beta - c^{i}_{2,1}\eta^{r}_{i,1}}{c^{i}_{2,2}}\right)\right\}$$

5.- Calculate:

$$\Phi\left[\left(\frac{\gamma_{j_{i,3}-1} - X'_{i,3}\beta - c^{i}_{3,1}\eta_{i,1} - c^{i}_{3,2}\eta_{i,2}}{c^{i}_{3,3}}\right) \leq \eta_{i,3} < \left(\frac{\gamma_{j_{i,3}} - X'_{i,3}\beta - c^{i}_{3,1}\eta_{i,1} - c^{i}_{3,2}\eta_{i,2}}{c^{i}_{3,3}}\right)\middle| \eta_{i,1} = \eta^{r}_{i,1}, \eta_{i,2} = \eta^{r}_{i,2}\right].$$

6.- Repeat these stages to complete ***T*** observations per individual. The evaluation of individual likelihood, for the simulated values, will be given by:

$$L^{r}_{i}(\cdot) = \Phi\left[\left(\frac{\gamma_{j_{i,1}-1} - X'_{i,1}\beta}{c^{i}_{1,1}}\right) \leq \eta_{i,1} < \left(\frac{\gamma_{j_{i,1}} - X'_{i,1}\beta}{c^{i}_{1,1}}\right)\right].$$
$$\Phi\left[\left(\frac{\gamma_{j_{i,2}-1} - X'_{i,2}\beta - c^{i}_{2,1}\eta^{r}_{i,1}}{c^{i}_{2,2}}\right) \leq \eta_{i,2} < \left(\frac{\gamma_{j_{i,2}} - X'_{i,2}\beta - c^{i}_{2,1}\eta^{r}_{i,1}}{c^{i}_{2,2}}\right)\right].$$
$$\Phi\left[\left(\frac{\gamma_{j_{i,3}-1} - X'_{i,3}\beta - c^{i}_{3,1}\eta^{r}_{i,1} - c^{i}_{3,2}\eta^{r}_{i,2}}{c^{i}_{3,3}}\right) \leq \eta_{i,3} < \left(\frac{\gamma_{j_{i,3}} - X'_{i,3}\beta - c^{i}_{3,1}\eta^{r}_{i,1} - c^{i}_{3,2}\eta^{r}_{i,2}}{c^{i}_{3,3}}\right)\right]....$$

7.- Repeat stages 1 to 6 ***R*** times. The simulated individual likelihood will be given by:

$$L_{i}(\cdot) = \frac{1}{R}\sum_{r=1}^{R} L^{r}_{i}(\cdot)$$

The simulated likelihood function (2.8) for all the individuals is obtained directly as:

$$L\left(\beta,\gamma,\Omega\middle|Y\right) = \sum_{i=1}^{N} \ln L_{i} \qquad (2.8)$$

The simulated log likelihood function can be maximised with regards to the parameters included in vector β and γ and matrix Ω, using the usual BFGS numerical optimisation methods Train, 2003]. Finally, once the estimators $\hat{\theta}^0$ have been obtained, the variance-covariance matrix can be directly obtained by inverting the Hessian evaluated in the maximum likelihood estimators obtained. On the other hand, with regards to the practical implementation of the algorithm, when optimising the simulated likelihood function we must remember the constraints with which the parameters have to comply (non-negative variance, positive semi-definite matrix Ω, and so on). To respect these constraints, the following modifications are made:

a) Maximise in relation to $\left|\sigma_{\alpha_i}\right|$ instead of $\sigma^2_{\alpha_i}$.

b) Re-parameterise the correlation coefficients as $\rho_i = \dfrac{\tau_i}{1+|\tau_i|}$ and maximise in relation to τ_i. This guarantees that $\forall i \quad \rho_i \in (-1,1)$.

c) Re-parameterise the thresholds as $\gamma_j = \gamma_{j-1} + \exp(\kappa_j)$ and optimise in relation to K_j. This guarantees that $0 \le \gamma_1 \le \cdots \le \gamma_{J-1}$ for all the values of K_j.

d) In some iterations, the eigenvalues of Ω can be negative for some values of the vector of parameters θ. In this case, a procedure is established to guarantee that the matrix is positive definite:

1) Calculate the eigenvalues Ω and the eigenvectors s_i of Ω.

2) Establish all the negative eigenvalues to a positive number near to zero, obtaining:

$$\begin{cases} \lambda_i^* = \lambda_i & if \quad \lambda_i \ge 0 \\ \lambda_i^* = 1.0^{-8} & if \quad \lambda_i < 0 \end{cases}$$

3) Multiply the eigenvectors $\boldsymbol{s_i}$ by their corrected eigenvalues λ^*_i. Order them in a matrix $\boldsymbol{B}^*$.

4) Normalise the row vectors of $\boldsymbol{B}^*$ to obtain unitary norm. We thus obtain the matrix $\boldsymbol{B}$ resulting from a procedure similar to Cholesky decomposition since $\Omega=\boldsymbol{B}\cdot\boldsymbol{B}'$ (indeed, if all the eigenvalues are positive, matrix $\boldsymbol{B}$ results from applying Cholesky decomposition to the initial matrix Ω).

This procedure obtains, for all the values of the parametric space, a positive definite matrix Ω similar to the initial matrix (in all the tests performed, the differences between the elements of the original matrix and the matrix resulting from the procedure were never more than two tenths). On the other hand, since we have to guarantee that the covariances do not exceed, in absolute value, the product of the typical deviations (for which the correlations will be less than one), we add a penalty term to the likelihood function for unfeasible values.

3. Application to estimate the country-risk for non-developed countries

Following Ciarrapico (1992), the traditional theoretical approach to country risk in economic literature can be classified in two groups described as informal and formal methods. The former develop indices and ratings based on subjective criteria from qualitative and quantitative information about the borrowing country. The subjectivity of such methods lies in the choice of variables to be considered, in how they are weighted and in the value judgments employed by analysts when establishing ratings for different countries. There are a large number of studies related to how these ratings work, their ability to forecast financial crises and their relation to the principal macroeconomic variables [Cantor and Packer, 1996; Feder and Uy, 1985; Lee, 1993] and their efficacy according to the degree of development of the country concerned [Erb et al, 1995]. On the other hand, the formal or statistical methods are based on estimating the likelihood of certain types of events involving economic-financial and political-administrative risk, using different statistical techniques.

This is exemplified by work such as [Blejer and Schumacher, 1998; Cornelius, 2000; Ferson and Harvey, 1999; Harvey and Zhou, 1993]. There are also two subdivisions in this group. The former is represented by studies aiming at determining country risk premiums and default probabilities from the market prices of public debt (models free-arbitrage opportunities), which evidently requires the existence of liquid markets for its negotiation; and the latter involves a rating and its associated probability of insolvency according to the micro and macro variables used to describe it. Different econometric techniques are used including, among others, qualitative dependent variable models. Our study fits into this last group because of two aspects: firstly, because the financial markets usually active negotiate the debt of developed countries and, secondly, because we intend to study only non-developed countries.

3.1. Formalisation of rating: Country risk as company risk. Country risk represents the solvency of all the counterparties belonging to the same geographical area, politically and legally defined as a State, either for temporary or permanent reasons. The purpose of estimating country risk is to establish which countries present problems which could affect operations either with the State or the Public Administration, or with private counterparties in those countries, and the severity of these problems. Once they are classified, and according to the risk rating assigned to them, investors will require earnings in proportion to the risk involved. The causes of a country's insolvency can be either economic-financial or political, so that risk is thus classified according to its origin as: 1) Sovereign Risk, corresponding to two possible events: on the one hand, a State's repudiation of its debt (total or partial); and on the other, deferral or restructuring of the debt, involving provisional non-payment with a subsequent renegotiation of the contractual conditions on the country's debt. 2) Transfer or Cash Risk, caused by the lack of sufficient means of payment to face foreign debt obligations. 3) Political or Administrative Risk, it corresponds to potential losses derived from a political and social change, and in a credit risk model it is represented by the likelihood of migration of worsening solvency conditions. To identify these risks with actual economic situations and a country's finances, we will follow the proposals of Blejer and Schumacher (1998) and Cornelius (2000) and establish a simile between a country's insolvency and a company's insolvency, by seeking equivalences between a company's financial situation, as revealed in its financial statements, and that of a country, according to the European System of National Accounts or SEC95 [Consejo Europeo, 1996]. In the first place, we have to find a concept in the country similar to business capital, since this is ultimately the guarantee that creditors will be paid. We thus define national capital as the sum of the capital possessed by resident individuals and companies, together with State capital and the capital of other public agencies. For that, the SEC95 defined a country's Annual Variation in Net Wealth (a concept similar to the balance of a company's profit and loss account) as the saving generated by its economy (S_t) after covering the consumption or depreciation of its productive goods ($CKfixed_t$) during said period, increased or decreased by the net balance of the capital transfers (TKN_t) performed with the rest of the world:

$$\Delta N_{t,t+1} \equiv S_t - CKfixed_t \pm TKN_t \qquad (3.1)$$

Then, we define the situation or risk rating[5] of country *i* at time *t* as $Y_{i,t}$, where *j* is the country risk rating for each situation, that is:

1.- Risk of repudiation (*j=3*):

$$Y_{i,t} = \begin{cases} = 3 & if \left[\Delta N_{t-1,t} < \Delta N_{t-2,t-1} < 0\right] \\ \neq 3 & otherwise \end{cases} \qquad (3.2)$$

2.- Risk of renegotiation: For the purpose of this study, we understand that the risk of renegotiating a country's external debt arises in the first period in which the economy generates negative flows, or immediately after that period when, in spite of consuming its own resources, it presents a positive relative variation from the previous period. In other words, in this cases the rating is *j=2*:

$$Y_{i,t} = \begin{cases} = 2 & if \begin{cases} \left[\Delta N_{t-1,t} < 0 < \Delta N_{t-2,t-1}\right] \\ or \\ \left[\Delta N_{t-2,t-1} < \Delta N_{t-1,t} < 0\right] \end{cases} \\ \neq 2 & otherwise \end{cases} \qquad (3.3)$$

3.- Transfer or cash risk: A country's cash risk is perceived as the situation in which the current value of the short-term external debt ($P^{c/p}{}_t$) for its entire consolidated economy, is greater than the current value of its international reserves (R_t) at time *t*. Therefore, if the rating assigned in this case is *j=1*, the country's cash risk is defined as:

[5] The proposed ratings in this paper, for each one of the categories of country risk, are tried only for non-developed countries, as we saw in the introduction. The debt of these countries have not enough liquidity in the markets and the disposable information about solvency of these ones, used to be limited. Therefore, to become a general model, including developed countries, previously it would require to establish a new rating adding the information referred to debt markets.

$$Y_{i,t} = \begin{cases} =1 & if \left[R_t < P_t^{c/p} \right] \\ \neq 1 & otherwise \end{cases} \tag{3.4}$$

Finally, the *j=0* rating is reserved for countries exempt from risk.

4.- Administrative risk or risk of political-social change: It is determined by the probability of the country's situation changing to another in which it is less solvent. This final component of country risk will therefore depend on:

$$0 \leq \Pr\left[Y_{i,t} > Y_{i,t-1} \middle| \mathsf{F}_t \right] \leq 1 \tag{3.5}$$

Where F_t is the information available at *t*, given that it will be necessary to know a priori the estimations for that moment in time of the macroeconomic variables explaining risk situations.

3.2. Sample. The sample selected for the empirical study comes from Base 2002 World Development Indicators published by the World Bank. Of a total of 207 countries initially analysed, the sample in our analysis comprises a total of 40 countries[6] grouped by geographical area[7] for the 1980-2000 period:

1. Africa: Cameroon, Central African Republic, Chad, Cote d'Ivore, Ghana, Kenya, Madagascar, Malawi, Mali, Mauritania, Niger, Nigeria and Senegal. 2. Central America: Costa Rica, Dominican Republic, Guatemala, Honduras, Jamaica, Mexico and Nicaragua.
3. South America: Argentina, Brazil, Chile, Ecuador, Peru, Uruguay and Venezuela RB. 4. Asia-Pacific: India, Indonesia, Korea Republic, Nepal, Pakistan, Philippines, Sri Lanka and Thailand. 5. Arabic Countries: Egypt, Jordan, Morocco, Tunisia and Turkey.

The explanatory variables selected in our study respond to the need to find different indicators justifying the evolution of the risk situation of the countries in the sample. These variables are: as an indicator of the economy's driving force, we used the Value Added

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[6] This reduction from 207 to 40 countries is due to the restriction of the variables required to define the proposed risk situations, to the lack of information for the rest of the countries initially used and to the selection of countries which could be classified as "non-developing", since it is precisely here where the estimation of country risk is most useful, given the lack of cash markets where their public debt is negotiated. Data frequency is annual and the panel is balanced.

[7] As we will see later, countries are grouped by geographical area to avoid over-parameterising the model.

of the Primary Sector (*VA1*), which includes farming, livestock breeding and fishing, the Value Added of the Secondary Sector (*VA2*), comprising industry and construction, and the Value Added of the Tertiary Sector (*VA3*) corresponding to the service sector; to analyze the cash-risk, we include the difference (*M32*) between *M3* (disposable money supply) and *M2* (money and quasi-money); Domestic Loans (*CD*) from the banking sector are included because they are an indicator of the indebtedness of a country's resident sectors; the total Private Sector Debt (*DP*), unlike the previous variable, would add external indebtedness, indicating how much of the risk of a State's agents has been transmitted to the Rest of the World; Bank Liquidity (*LIQ*), measured as the ration between the current accounts held by banks in Central Banks and their total assets; Inflation (*G*), this indicator enables us to determine how the price level can have a negative impact on the solvency of all the agents comprising a State; the annual Exchange Rate (*TC*) of local currency with the United States dollar. This variable will indicate the expectations of the international financial markets concerning the economy in question, and; an economy's Net Capital Flows (*FN*) to the exterior. Given that most of the economic variables present heavy trends, and are not therefore stationary, the usual statistical inference is not applicable. In order to homogenise the information on the variables used, considering that they are not stationary, we have expressed them as relative variation rates, thus losing the first value of the sample, except for variables which are rates by definition (inflation).

3.3. Results. To select the independent variables to be included in each type of model (which evidently do not necessarily have to coincide with the same regressors in each model) and maximise the corresponding simulated likelihood, we go from a general to specific approach in the following stages. First, start in the type 1 model (T1); second, establish a minimum and maximum number of lags in the independent variables (in our case, 1 and 2 respectively[8]) and include all the regressors in the model; third, maximise the simulated

[8] Contemporary regressors are not included because, when the rating for a year is calculated, the data for the current year is not usually available.

likelihood function[9], as described in the previous section, and compare the individual significance of each parameter; forth, eliminate the variables of which the parameters are not significant on a 10% level and return to step 3; and fifth, if all the variables are significant, go on to the next type of model and start again at step 2, and if the type is 7, end the process. The regressors which were finally significant for each model, and there corresponding beta values, are shown on table 1. When analysing the table 1, we see that in the type 1 model, the significant regressors are different from those in the rest of the models.

Table 1. Estimated values of β

Regressors	Type-1	Type-2	Type-3	Type-4	Type-5	Type-6	Type-7
Cte.	0.3409 (7.17)	0.5195 (3.18)	0.407 (2.59)	0.4189 (2.38)	0.7477 (3.86)	0.4495 (2.49)	0.3528 (1.47)
DP t-2			0.1441 (1.52)		0.1277 (1.37)	0.1417 (1.44)	
G t-2	0.0116 (1.54)						
FN t-2			0.0136 (1.64)		0.0131 (1.56)	0.0119 (1.40)	
VA2 t-1		-0.4767 (-1.68)	-0.5877 (-2.15)	-0.4820 (-1.70)	-0.5999 (-2.18)	-0.6046 (-2.15)	-0.5107 (-1.78)
M32 t-1		-0.0525 (-1.91)	-0.0694 (-2.91)	-0.0525 (-1.91)	-0.066 (-2.68)	-0.0681 (-2.76)	-0.0667 (-2.47)
DP t-1	-0.2558 (-2.19)						
LIQ t-1		0.0923 (2.52)		0.0932 (2.55)			0.0638 (1.75)
G t-1	0.0177 (2.26)						

Columns Type-1 to Type-7 report the estimations by maximum likelihood of β_k in equation (2.4). For each type, we only include the regressors selected in the model. The (t-Test) record the values of the individual significance test statistics.

[9] As usual with simulation methods, the same random values have been generated in each of the iterations in the optimisation process with the BFGS.

This means that, when working with cross section discrete variable models, the choice of explanatory variables could be mistaken with respect to what would happen when considering random and time effects derived from the panel data. It also appears that the inflation variation rates at *t-1* and *t-2* are the variables showing part of the random effect and time correlation not assumed in the type 1 model. In the types of model contemplating time autocorrelation[10] (3, 5 and 6), the regressors are the same and also more numerous than in the other types.

On the other hand, they are the only models presenting a significant variable at *t-2*, Private Debt. Finally, in the types of model including random effects (2, 4 and 7), the regressors also coincide but in this case they only include one lag (*t-1*). We can conclude, therefore, that regressor selection will depend on the structure of the variance-covariance matrix chosen, so a correct procedure would involve using a likelihood ratio test to choose the most appropriate type of model for the data in order to determine the explanatory variables. Finally, the significant regressors in all the models (except type 1) are Value Added of the Secondary Sector at *t-1* and the M3-M2 differential (*M32*), also at *t-1*; the former would indicate how the country is progressing with regards to ratings 2 and 3, whereas the appearance of the latter would be related to cash availability, the problem contemplated in rating 1. Since, in this type of model, the sign of the beta parameter does not indicate the degree of influence of the corresponding regressor on the probability of a certain rating, to measure this marginal effect we have to calculate the derivative of this probability on each of the independent variables chosen. This derivative is calculated as:

$$\begin{cases} \dfrac{\partial \Pr\left(Y_{i,t}=j\right)}{\partial x_k} = -\phi\left(\dfrac{X'_{i,t}\beta}{\sigma_{\varepsilon_{i,t}}}\right)\dfrac{\beta_k}{\sigma_{\varepsilon_{i,t}}} & if \quad j=0 \\ \dfrac{\partial \Pr\left(Y_{i,t}=j\right)}{\partial x_k} = \left[\phi\left(\dfrac{\gamma_{j-1}-X'_{i,t}\beta}{\sigma_{\varepsilon_{i,t}}}\right) - \phi\left(\dfrac{\gamma_j - X'_{i,t}\beta_k}{\sigma_{\varepsilon_{i,t}}}\right)\right]\dfrac{\beta_k}{\sigma_{\varepsilon_{i,t}}} & if \quad j=1,2,\cdots,J-2 \\ \dfrac{\partial \Pr\left(Y_{i,t}=j\right)}{\partial x_k} = \phi\left(\dfrac{\gamma_j - X'_{i,t}\beta}{\sigma_{\varepsilon_{i,t}}}\right)\dfrac{\beta_k}{\sigma_{\varepsilon_{i,t}}} & if \quad j=J-1 \end{cases} \tag{3.6}$$

[10] Except in the type 7 model in which, although it includes autocorrelation, the regressors do not coincide with the rest.

where $\Phi(\cdot)$ represents the density function of the normal standard[11] evaluated at the corresponding point. The previous equation shows that, before an increase in a variable x_k, the effect on the probability of each rating would have the same sign as parameter β_k for ratings 0 and *J-1*.

Table 2. Derivatives of each rating in relation to each regressor.

Models	Rating	DP t-2	G t-2	FN t-2	VA2 t-1	M32 t-1	DP t-1	LIQ t-1	G t-1
Type 1	0		-0.0044				0.0960		-0.0067
	1		0.0028				-0.0622		0.0043
	2		0.0010				-0.0220		0.0015
	3		0.0005				-0.0118		0.0008
Type 2	0				0.1287	0.0142		-0.0249	
	1				-0.0815	-0.0090		0.0158	
	2				-0.0287	-0.0032		0.0056	
	3				-0.0185	-0.0020		0.0036	
Type 3	0	-0.0342		-0.0032	0.1395	0.0165			
	1	0.0214		0.0020	-0.0873	-0.0103			
	2	0.0081		0.0008	-0.0329	-0.0039			
	3	0.0047		0.0005	-0.0193	-0.0023			

For each type of model, the derivatives measure the change in the probability of each rating when variable x_k increases (equation 3.6.). For each type, we only include the derivative in relation to the significant regressors in this model. In the more complex types of model (4 to 7), the derivative recorded is per geographical area, see tables 2b and 2c in annex.

However, in all the intermediate ratings (j = 1, …, *J-2*), the effect of the probability of the rating of an increase in variable x_k does not have to coincide with the sign of β_k. Logically, for the sum of the probabilities to be one, the sum of the derivatives must be zero, since the changes in the probability of each rating must be compensating to continue to add up to one. Furthermore, note that the derivative in

[11] Remember that, in general, the variance of disturbance $\varepsilon_{i,t}$ will not be one, so typification is required for the distribution to be normal standard.

relation to a variable x_k depends on the value of all the regressors through vector $X_{i,t}$. The derivative, therefore, will be different when the values of the independent variables change. Table 2 summarizes all the derivatives calculated in each of the models. To chose a value representative of the vector of variables $X_{i,t}$, it is calculated as the global mean vector $\bar{X}$ in types 1, 2 and 3, whereas the mean per group of countries $\bar{X}_i$ is chosen in the other types.

Analysing the results by model, in type 1 the variable with the greatest influence is Private Debt at *t-1*. Its effect is positive for rating 0 and negative for the rest. However, an increase in private debt should not improve the rating. One possible explanation could be a poor choice of regressors in this type of model. For types 2, 3, 4, 5, 6 and 7, the variable with the greatest influence is the rate of variation of the Value Added of the Secondary Sector at *t-1*. Furthermore, this variable logically has a positive impact on rating 0 and a negative impact on the rest.

In other words, with increased growth of Industrial Value Added a rating of 0 is more likely than a poorer rating. For models 4, 5, 6 and 7, in which there is a division by geographical group[12], we can add that, in type 4, the group most influenced by this regressor was Asia-Pacific. In type 5 it was Africa, in group 6 influence was similar in all the groups, and in type 7 the group with most influence was again Asia-Pacific. These results seem logical, since it is precisely in the Asia-Pacific area where there is a larger proportion of industrial production than in the other geographical areas studied.

Analysing the results obtained, in the estimations of the thresholds γ_j on table 3 we can observe that, as the covariance matrix becomes more complex, the value of the threshold increases. The threshold columns record the value of $.\gamma$ in equation (2.5.). The value of Threshold-1 is always 0 due to normalisation. The t-Tests compare the individual significance of K_j in the expression: $\gamma_j = \gamma_{j-1} + \exp(\kappa_j)$. We could therefore conclude that there is a direct relationship between the inclusion of random and time effects and the threshold

[12] For type 6 we have also calculated the derivatives evaluating $\boldsymbol{X_{i,t}}$ on global mean vector $\bar{X}$. However, the results obtained are practically identical to the mean of the derivatives calculated separately for each group.

values. This relationship is most important in the case of time correlations.

Table 3. Values of γ estimated in each type of model

Models	Threshold-1	Threshold-2	t-Test	Threshold-3	t-Test
Type 1	0.0000	1.8373	14.7097	2.428	-3.4168
Type 2	0.0000	2.5231	20.0828	3.2729	-1.8977
Type 3	0.0000	2.7103	20.9937	3.6605	-0.3282
Type 4	0.0000	2.5222	20.0430	3.2762	-1.8620
Type 5	0.0000	2.7188	21.2585	3.6534	-0.4369
Type 6	0.0000	2.7566	6.7748	3.7105	-0.3062
Type 7	0.0000	2.8232	21.8502	3.7281	-0.6488

Although the third threshold is not significant in some types of model (especially when time correlation is included), this is due to the re-parameterisation carried out when optimising. Actually, threshold 2 and 3 are parameterised as $\gamma_j = \gamma_{j-1} + \exp\left(\kappa_j\right)$ and the t-test significance statistic refers to parameter K_j of the exponential, so if it is not significant, it is indicating that the difference from the previous threshold is not statistically different from the unit. Typical deviations are not included in the first threshold since, for all models, it is normalised in the null value. For the number of parameters not to be excessive in types 4, 5 and 7, the countries are grouped into 5 geographical areas (Africa, Central America, South America, Asia-Pacific and Arabic Countries) so that the individual parameters are the same for all the countries in the same area.

This considerably reduces the number of individual parameters to be estimated and, if countries in the same area behave similarly, does not represent a significant loss of generality, contemplating the diversity associated to different geographical regions. So, tables 4 and 5 summarise the estimations of the parameters of the variance-covariance matrix Ω. Table 4 shows the typical deviations σ_{α_i} of the random effects. In the type 2 model (typical deviation common to all the groups), the estimation is significant and slightly below 1 (0.9575). On the other hand, when in type 4 the random effect is permitted to be different for each geographical area, the greatest typical deviation is 1.2099, corresponding to South America, whereas the lowest is Asia-pacific with 0.4789. If, however, in addition to the random effect we include time correlation (types 6 and 7), we find that in type 6, with the same random effect and autocorrelation for all the groups, the standard deviation of the random effect is lower (0.8777) than in type 2.

Table 4. Estimated values of σ_α

Areas	Type 1	Type 2	Type3	Type 4	Type 5	Type 6	Type 7
Africa	1	0.9575 (7.91)	1	1.0046 (4.24)	1	0.8777 (5.21)	1.1430 (3.91)
Central America	1	0.9575 (7.91)	1	1.0736 (3.53)	1	0.8777 (5.21)	6.92E-07 (6.12E-07)
South America	1	0.9575 (7.91)	1	1.2099 (3.40)	1	0.8777 (5.21)	0.8662 (0.55)
Asia-Pacific	1	0.9575 (7.91)	1	0.4789 (2.71)	1	0.8777 (5.21)	6.81E-08 (1.99E-07)
Arabic Country	1	0.9575 (7.91)	1	0.8406 (2.61)	1	0.8777 (5.21)	0.6332 (0.71)

Columns Type-1 to Type-7 record the typical deviations of the random effects in the disturbances. The (t-statistic) is not included in types 1, 2 and 5 because these models do not include random effects. In types 2 and 6, the value of σ_α is the same for all the individuals.

Therefore, it at least partly appears that the heterogeneity contemplated by the random effect in type 2 will be included in the time correlation. In the type 7 model, in which different random effects and autocorrelation are permitted for each group, we see that the greatest typical deviation of the random effect is 1.143 in Africa, the only group in which it was significant. To summarize, when time correlation is added, the importance of the random effect decreases because, to a large extent, the current rating is explained by the situation at the previous moment in time. Only in Africa does the random effect remain significant, which is logical considering the different development of this group in relation to the rest.

Table 5 summarizes the estimations of the parameters ρ_i including time correlations. When analyzing the table, we detect a significant positive autocorrelation in all the types. For example, in type 3 (parameter ρ the same for all the groups), the estimation is 0.79; however, in type 5 (parameter ρ_i different for each group), the estimations obtained are around this mean value, with the lowest in Africa (0.65) and the highest in South America (0.883).

Table 5. Estimated values of ρ

Areas	Type-1	Type-2	Type-3	Type-4	Type-5	Type-6	Type-7
Africa	0.00	0.00	0.7910 (6.37)	0.00	0.6546 (3.42)	0.6823 (4.64)	0.3532 (2.38)
Central America	0.00	0.00	0.7910 (6.37)	0.00	0.8310 (2.65)	0.6823 (4.64)	0.8384 (2.62)
South America	0.00	0.00	0.7910 (6.37)	0.00	0.8833 (2.29)	0.6823 (4.64)	0.8190 (0.76)
Asia-Pacific	0.00	0.00	0.7910 (6.37)	0.00	0.7869 (2.68)	0.6823 (4.64)	0.7507 (3.05)
Arabic country	0.00	0.00	0.7910 (6.37)	0.00	0.8823 (1.67)	0.6823 (4.64)	0.7891 (1.30)

Columns Type-1 to Type-7 record the autoregressive parameters in the disturbances. The (t-statistic) is not included in types 1, 2 and 4, because these models do not include time correlations. In types 3 and 6, the value of ρ is the same for all the individuals.

On the other hand, when random effects and autocorrelation parameters common to all the groups are included (type 6), the estimated parameter decreases (0.68), as expected, since part of the performance over time is considered in the random effects. However, in type 7, which permits different random effects and time correlation for each group, the lowest is in the group with the only significant random effect (Africa), where the parameter is 0.35, whereas the highest value (0.84) corresponds to Central America. Both in South America and the Arabic countries, autocorrelation is not significant, probably because of its highly volatile ratings in the sample period. Once all the parameters have been estimated, it is simple to obtain the probability of each rating, given the values of $\boldsymbol{X}_{i,t}$. Table 6 shows these probabilities choosing the $\boldsymbol{X}_{i,t}$ values corresponding to the last available period[13]. To calculate these

[13] In types 4, 5 and 7, the parameters estimated for each country correspond to those of the relevant group or geographical area.

probabilities, described as unconditional, we only consider the history of previous ratings. This probability is calculated as:

$$
\begin{aligned}
&\Pr\left(Y_{i,t} = 0\right) = \Phi\left(\varepsilon_{i,t} < \frac{-X_{i,t}{}'\beta}{\sigma_{\varepsilon_i}}\right)\\
&\vdots\\
&\Pr\left(Y_{i,t} = j\right) = \Phi\left(\frac{\gamma_{j-1} - X_{i,t}{}'\beta}{\sigma_{\varepsilon_i}} \le \varepsilon_{i,t} < \frac{\gamma_{j} - X_{i,t}{}'\beta}{\sigma_{\varepsilon_i}}\right)\\
&\vdots\\
&\Pr\left(Y_{i,t} = J\right) = 1 - \Phi\left(\frac{\gamma_{J-1} - X_{i,t}{}'\beta}{\sigma_{\varepsilon_i}} \le \varepsilon_{i,t}\right)
\end{aligned}
\tag{3.7}
$$

We had selected a country of each group or geographical area to estimate these unconditional and conditional probabilities. If we examine the results[14] of table 6, we see that, as expected, the greatest probability is concentrated in ratings 0 and 1, whereas generally the smallest probability corresponds to rating 3. Obviously, the sum of all the probabilities must be 1. However, the previous probabilities do not consider the country's history, so we do not use the information provided by each one's time correlations. For example, we have calculated the probabilities of each rating in period *T+1* for different countries from different groups, considering all the information available up to period *T* (conditional probability).

These probabilities can be calculated from the estimated parameters using the GHK as described in the previous subsection. The results (using the same vector $\boldsymbol{X_{i,T+1}}$ as for the conditional probabilities) are shown on table 7. This table 7 shows significant differences between the probabilities obtained for the different types of model[15]. This is particularly relevant is we compare the probabilities obtained with type 1 (process similar to cross-section probit) with the rest of the models, confirming the need to consider both the random effects of heterogeneity and time correlations.

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[14] We only include one representative country from each group or geographical area, although the other countries are available. Contact either of the authors by e-mail.

[15] Obviously, if we compare the conditional and unconditional probabilities, they are the same in Type 1 in which matrix Ω is identity.

Table 6. Unconditional Probabilities

Models	Rating	Brazil	Egypt	India	México	Nigeria
Type-1	0	0.3876	0.3759	0.3795	0.3620	0.3834
	1	0.5520	0.5600	0.5576	0.5691	0.5549
	2	0.0443	0.0468	0.0460	0.0499	0.0452
	3	0.0161	0.0174	0.0170	0.0190	0.0165
Type-2	0	0.3623	0.3797	0.3627	0.3785	0.4367
	1	0.5669	0.5556	0.5667	0.5563	0.5152
	2	0.0486	0.0449	0.0486	0.0452	0.0344
	3	0.0221	0.0198	0.0221	0.0200	0.0137
Type-3	0	0.4213	0.4304	0.4063	0.4184	0.4872
	1	0.5065	0.5005	0.5161	0.5084	0.4608
	2	0.0516	0.0496	0.0550	0.0522	0.0383
	3	0.0206	0.0195	0.0226	0.0210	0.0136
Type-4	0	0.4027	0.4025	0.3641	0.4120	0.4670
	1	0.5105	0.5514	0.6089	0.5208	0.4881
	2	0.0540	0.0342	0.0224	0.0451	0.0320
	3	0.0328	0.0119	0.0046	0.0222	0.0129
Type-5	0	0.3770	0.3847	0.3273	0.3538	0.3851
	1	0.4548	0.4533	0.5634	0.5185	0.5760
	2	0.0874	0.0851	0.0739	0.0790	0.0322
	3	0.0809	0.0770	0.0354	0.0488	0.0068
Type-6	0	0.4107	0.4205	0.3957	0.4085	0.4788
	1	0.5186	0.5121	0.5282	0.5200	0.4710
	2	0.0509	0.0487	0.0544	0.0514	0.0373
	3	0.0198	0.0186	0.0218	0.0201	0.0129
Type-7	0	0.4337	0.4459	0.4164	0.4408	0.4940
	1	0.4667	0.4847	0.5346	0.4769	0.4692
	2	0.0595	0.0466	0.0369	0.0525	0.0279
	3	0.0402	0.0229	0.0122	0.0299	0.0090

Probability of each rating calculated from the information available at each moment in time without considering prior history (equation 3.7.).

Table 7. Conditional Probabilities

Models	Rating	Brazil	Egypt	India	Mexico	Nigeria
Type-1	0	0.38761	0.37586	0.37946	0.36201	0.38339
	1	0.55203	0.56000	0.55758	0.56911	0.55492
	2	0.04428	0.04678	0.04600	0.04988	0.04517
	3	0.01608	0.01736	0.01696	0.01900	0.01653
Type-2	0	0.33656	0.45794	0.76092	0.25430	0.36009
	1	0.64134	0.53215	0.23817	0.70715	0.62086
	2	0.01899	0.00877	0.00085	0.03213	0.01646
	3	0.00312	0.00113	0.00006	0.00642	0.00259
Type-3	0	0.71231	0.83454	0.81893	0.75893	0.27740
	1	0.28665	0.16500	0.18052	0.24028	0.68387
	2	0.00100	0.00045	0.00053	0.00077	0.03387
	3	0.00003	0.00001	0.00002	0.00003	0.00485
Type-4	0	0.33534	0.45932	0.79855	0.25270	0.35968
	1	0.64211	0.53075	0.20081	0.70789	0.62099
	2	0.01938	0.00881	0.00061	0.03286	0.01672
	3	0.00317	0.00112	0.00004	0.00655	0.00261
Type-5	0	0.73034	0.87732	0.86327	0.77681	0.24313
	1	0.26872	0.12234	0.13634	0.22249	0.69994
	2	0.00091	0.00033	0.00038	0.00069	0.04776
	3	0.00003	0.00001	0.00001	0.00002	0.00918
Type-6	0	0.67246	0.79655	0.84863	0.68508	0.27388
	1	0.32623	0.20289	0.15108	0.31355	0.69240
	2	0.00127	0.00054	0.00028	0.00132	0.02986
	3	0.00005	0.00002	0.00001	0.00005	0.00386
Type-7	0	0.71680	0.86266	0.88030	0.76705	0.25660
	1	0.28245	0.13708	0.11951	0.23238	0.70121
	2	0.00072	0.00025	0.00019	0.00055	0.03568
	3	0.00003	0.00001	0.00001	0.00002	0.00651

Probability of each rating calculated from all the information available in each period and considering prior history. To calculate these probabilities, we have to apply the GHK simulator to the observed history and simulate the likelihood for period $T+1$ for each raring, given $X_{i,T+1}$. The values of $X_{i,T+1}$ used are the same as for unconditional probability.

This is also evident when we perform likelihood tests to choose between different models. In our case, these tests are the results of applying linear constraints to the variance-covariance matrix Ω, since each model arises as a particular case from another when we limit the set of parameters. For example, if in the type 7 model (the most general possible) we establish the following null hypothesis: Ho: $\sigma_{\alpha 1} = \sigma_{\alpha 2} = \ldots = \sigma_{\alpha N}$ and $\rho_1 = \cdots = \rho_N$, we obtain the type 6 model, whereas if we consider Ho: $\sigma_{\alpha 1} = \sigma_{\alpha 2} = \ldots = \sigma_{\alpha N} = 0$, we obtain the type 5 model.

Table 8. Likelihood ratio, LR, tests for model selection.

Models	Constraints	LR Model Type-7	p-value
Type-1	10	419.932	0.000
Type-2	9	169.644	0.000
Type-3	9	28.660	0.001
Type-4	5	165.344	0.000
Type-5	5	20.248	0.001
Type-6	8	20.748	0.008
Models	Constraints	LR Model Type-6	p-value
Type-1	2	399.184	0.000
Type-2	1	148.896	0.000
Type-3	1	7.912	0.005
Models	Constraints	LR Model Type-5	p-value
Type-1	5	399.684	0.000
Type-3	4	149.396	0.000
Models	Constraints	LR Model Type-4	p-value
Type-1	5	254.588	0.000
Type-2	4	4.300	0.367
Models	Constraints	LR Model Type-3	p-value
Type-1	1	391.272	0.000
Models	Constraints	LR Model Type 2	p-value
Type-1	1	250.288	0.000

Likelihood ratio statistics are calculated as $-2(\ln L_R - \ln L_{NR})$ where L_R and L_{NR} represent the values of the restricted and non-restricted likelihood logarithm, respectively. The p-value column shows the p-values calculated from distribution χ^2_m where m is the number of constraints.

Table 9. Likelihood, Information Criteria and Computational Time

Models	Log-Likel.	AIC	BIC	Comp. Time
Type-1	-694.316	1400.63	1428.43	0:49:30
Type-2	-569.172	1152.34	1184.78	2:42:38
Type-3	-498.680	1013.36	1050.43	2:57:15
Type-4	-567.022	1156.04	1207.01	10:04:11
Type-5	-494.474	1012.95	1068.55	13:29:06
Type-6	-494.724	1007.45	1049.15	4:02:24
Type-7	-484.350	1000.70	1074.83	21:23:13

The Log-Likel. column records the log likelihood values simulated at the optimum point (equation 2.8.). Calculated on a Pentium IV computer with 3.0 GHz processor. Total time: 55:28:17

Obviously, the type 1 model is the most restricted case possible and included (arising as a particular case) from the rest of the models. Table 8 shows the results of performing likelihood comparisons between more general models and more restricted cases. Rejection of the null hypothesis (p-value beneath 0.05) indicates that the most general model is preferable to the restricted case. If we analyse the results, we see that, in practically every case, the more general models (with greater complexity in matrix Ω) are substantially better than the more restricted cases. The only exception is in type 4 (different random effects) compared with type 2 (single random effect), indicating that the degree of heterogeneity, measured by σ_α, can be similar between the different groups of countries considered. As usual, the type 1 model shows greater rejection in all the cases studied.

4. Conclusions

This paper presents a method for estimating country credit ratings aimed at avoiding three problems which mar arise on today's markets: in the first place, it avoids agency rating systems considered to be "black boxes"; secondly, it avoids the problem of measuring country risk if debt is not negotiated on cash markets; and finally, it is an advanced method for the internal measurement of country risk from the perspective of the recent Basel Capital Accord. This study introduces two main aspects: The first is the approach used to define country risk situations. It is an adaptation of other work performed on business solvency. The second is the model for estimating the probability of each possible rating. This model (Ordered Probit on panel data) enables us to contemplate different variance-covariance matrix structures, giving rise to the possibility of including time correlation, random effects or both. We used the GHK simulator to estimate the model, because in view of the impossibility of optimising the exact likelihood function, we decided to work with the simulated likelihood function. All the above has been completed with a practical implementation of the model on a sample of 40 non-developing countries, during the 1980-2000 period, obtained from the Base 2002 of World Development Indicators published by the World Bank and grouped into 5 geographical areas (Africa, Central America, South America, Asia-pacific and Arabic Countries), under the hypothesis of similar behaviour in each group. The principal results obtained include the following:

1) The choice of variables explaining the ratings is different for the type 1 model, which does not consider either individual heterogeneity effects or time dependence, in relation to the other models which do contemplate heterogeneity and autocorrelation. Since the ratings published by agencies usually only consider the values of variables explaining the ratings and not the dependence structure, these ratings may be biased.On the other hand, the regressor explaining the different ratings for all except the cross section model (type 1 model) was the annual rate of variation of the Value Added of the Secondary Sector. Therefore, it seems clear that greater industrial development is the fundamental variable explaining a better rating. The thresholds distinguishing between different non-observable utility levels, which in turn generate the probability of each rating, are a direct function of the complexity of the covariance matrices of the disturbances.

2) In likelihood ratio tests and when analysing information criteria (AIC and BIC), the preferred models are always those which propose more complex covariance matrices, so we conclude that default probability models should include heterogeneity, time autocorrelation and any other characteristic adding value to the model, even more than the search for a large number of regressors. This confirms the need for caution with the usual rating procedures,

since they are usually based on cross section data and ignore the importance dependencies in the panel data.

3) The proposed models enable us to determine the probability of each rating for each individual, both unconditionally and conditionally. To illustrate this, we perform a simulation with a set of 5 countries (Brazil, Egypt, India, Mexico and Nicaragua), each one representing one of the 5 groups or geographical areas (South America, Arabic countries, Asia-Pacific, Central America and Africa, respectively). Logically, the unconditional probability of an individual having one rating or another at any given time is the same as the conditional probability, considering all the information available as of that time, in the type 1 model. On the other hand, this unconditional probability differs significantly from the conditional probabilities for the other types which, again, leads us to question the accuracy of the usual ratings based exclusively on cross section data. This is particularly relevant in the short and medium terms since, for the models including time correlation, the difference between the two probabilities is especially significant for 1, 2 or 3-year horizons.

Several effects are observed when analysing the conditional probabilities of types 2 to 7: 1) When the model contemplates time correlation (types 3, 5, 6 and 7), the greatest conditional probability corresponds to the previous rating. In other words, the greatest probability is that the situation will remain unaltered. 2) However, if the model contemplates random effects of individual heterogeneity (types 2 and 4), the greatest conditional probability corresponds to the next worst rating. In other words, the greatest probability is that the situation will worsen. 3) Nigeria is an exception because, regardless of the chosen type, the greatest probability is that the situation will worsen to the next rating; this merely confirms our results, since Africa was the group with most heterogeneity (the random effect is greater and more significant).

To summarise, we can conclude that the greater the heterogeneity, the greater the probability of migration or a change to the worse in the rating. Finally, one possible related line of research would be to include cross-correlation between individuals to permit the transmission of shocks between different areas and observe their impact on the different ratings.

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Annex: Derivatives of each rating in relation to each independent variable.

Models Type-4 and Type-5

Model	Rating	VA2_1	M32_1	LIQ_1	Model	Rating	DP_2	FN_2	VA2_1	M32_1
T-4 Gr.1	0	0.1299	0.0142	-0.0251	T-5 Gr.1	0	-0.0332	-0.0034	0.1559	0.0172
	1	-0.0847	-0.0092	0.0164		1	0.0208	0.0021	-0.0979	-0.0108
	2	-0.0273	-0.0030	0.0053		2	0.0090	0.0009	-0.0424	-0.0047
	3	-0.0178	-0.0019	0.0034		3	0.0033	0.0003	-0.0155	-0.0017
T-4 Gr.2	0	0.1265	0.0138	-0.0245	T-5 Gr.2	0	-0.0261	-0.0027	0.1227	0.0135
	1	-0.0810	-0.0088	0.0157		1	0.0108	0.0011	-0.0508	-0.0056
	2	-0.0266	-0.0029	0.0052		2	0.0078	0.0008	-0.0366	-0.0040
	3	-0.0189	-0.0021	0.0037		3	0.0075	0.0008	-0.0353	-0.0039
T-4 Gr.3	0	0.1187	0.0129	-0.0230	T-5 Gr.3	0	-0.0225	-0.0023	0.1056	0.0116
	1	-0.0697	-0.0076	0.0135		1	0.0070	0.0007	-0.0327	-0.0036
	2	-0.0262	-0.0029	0.0051		2	0.0061	0.0006	-0.0288	-0.0032
	3	-0.0227	-0.0025	0.0044		3	0.0094	0.0010	-0.0442	-0.0049
T-4 Gr.4	0	0.1636	0.0178	-0.0317	T-5 Gr.4	0	-0.0285	-0.0029	0.1339	0.0147
	1	-0.1368	-0.0149	0.0265		1	0.0138	0.0014	-0.0651	-0.0072
	2	-0.0211	-0.0023	0.0041		2	0.0086	0.0009	-0.0403	-0.0044
	3	-0.0057	-0.0006	0.0011		3	0.0061	0.0006	-0.0286	-0.0031
T-4 Gr.5	0	0.1406	0.0153	-0.0272	T-5 Gr.5	0	-0.0226	-0.0023	0.1061	0.0117
	1	-0.1016	-0.0111	0.0197		1	0.0071	0.0007	-0.0334	-0.0037
	2	-0.0262	-0.0029	0.0051		2	0.0062	0.0006	-0.0290	-0.0032
	3	-0.0129	-0.0014	0.0025		3	0.0093	0.0010	-0.0438	-0.0048

Models Type-7 and Type-6

Model	Rating	VA2_1	M32_1	LIQ_1	Model	Rating	DP_2	FN_2	VA2_1	M32_1
T-7 Gr.1	0	0.1272	0.0166	-0.0159	T-6 Gr.1	0	-0.0336	-0.0028	0.1435	0.0162
	1	-0.0902	-0.0118	0.0113		1	0.0212	0.0018	-0.0905	-0.0102
	2	-0.0245	-0.0032	0.0031		2	0.0079	0.0007	-0.0337	-0.0038
	3	-0.0125	-0.0016	0.0016		3	0.0045	0.0004	-0.0192	-0.0022
T-7 Gr.2	0	0.1094	0.0143	-0.0137	T-6 Gr.2	0	-0.0336	-0.0028	0.1434	0.0162
	1	-0.0659	-0.0086	0.0082		1	0.0212	0.0018	-0.0903	-0.0102
	2	-0.0239	-0.0031	0.0030		2	0.0079	0.0007	-0.0338	-0.0038
	3	-0.0196	-0.0026	0.0025		3	0.0045	0.0004	-0.0193	-0.0022
T-7 Gr.3	0	0.1032	0.0135	-0.0129	T-6 Gr.3	0	-0.0335	-0.0028	0.1430	0.0161
	1	-0.0573	-0.0075	0.0072		1	0.0209	0.0018	-0.0892	-0.0101
	2	-0.0232	-0.0030	0.0029		2	0.0080	0.0007	-0.0342	-0.0038
	3	-0.0227	-0.0030	0.0028		3	0.0046	0.0004	-0.0196	-0.0022
T-7 Gr.4	0	0.1318	0.0172	-0.0165	T-6 Gr.4	0	-0.0336	-0.0028	0.1436	0.0162
	1	-0.0980	-0.0128	0.0123		1	0.0213	0.0018	-0.0908	-0.0102
	2	-0.0234	-0.0031	0.0029		2	0.0079	0.0007	-0.0337	-0.0038
	3	-0.0105	-0.0014	0.0013		3	0.0045	0.0004	-0.0191	-0.0022
T-7 Gr.5	0	0.1146	0.0150	-0.0143	T-6 Gr.5	0	-0.0336	-0.0028	0.1432	0.0161
	1	-0.0728	-0.0095	0.0091		1	0.0210	0.0018	-0.0897	-0.0101
	2	-0.0244	-0.0032	0.0031		2	0.0080	0.0007	-0.0340	-0.0038
	3	-0.0175	-0.0023	0.0022		3	0.0046	0.0004	-0.0195	-0.0022